# Trade with differences in technologies

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# Abstract

According to the factor price equalization theorem, trade in goods itself would result in the equalization of factor prices across countries when we do not have factor intensity reversal. In this paper, using two countries, two goods and two factors of production, I have shown that if the technologies differ across countries, then the factor prices will not get equalized after trade and in my model there is no possibility of factor intensity reversal. Under certain conditions, a capital abundant country will export a labor intensive good. Trade could increase the factor price difference across the countries. Perfect capital mobility across the countries is not going to result in the equalization of wage rate across the countries.

Introduction

International trade is one of the most interesting and controversial topic in today's world of economics. Throughout the ages, it has shown its importance for any country which has any form of economic relationship with the rest of the world. It is not surprising that some people termed it as "engine of growth" for certain period of time in the past. In the modern world, every country is involved in trade with the rest of the world. The next thing is the obvious and important question about trade across countries is what is the basis of trade or what determines the pattern of trade?

Many economists tried to answer this question and they came up with significantly

different answers. These theories about international trade vary over time. Let us try to

look at some of the prominent theories that has dominated this field and shaped the

thinking and behavior of economists in particular and the policy makers.

#### Literature Review

Adam Smith in his famous book "Wealth of the Nations" (1776) came up with the theory of absolute advantage and according to him a country can export the good only when it has an absolute advantage in the production of that good. The problem with this theory arises when a country does not have absolute advantage in the production of any good and in that case according to Smith's theory, it will not be able to

export any good. Robert Torrens in 1815 wrote an essay in which he pointed out that it was to England's advantage to trade other goods with Poland in return for corn, even though it might be possible to produce corn cheaply in England than in Poland. Ricardo in his book "The principles of political economy and taxation" (1817) provided a systematic explanation for this and this theory was named as "Theory of comparative advantage". In his theory he assumed that there are two countries, two goods and one factor of production. He assumed that there is full employment in the economies and we have constant opportunity costs, perfect mobility of factors of production within countries but it is immobile across countries. This theory would remain valid even if you extend the number of goods and allow for the possibility of more than one factor of production. The key factor in this theory is the differences in the productivity which results in differences in the opportunity costs.

Heckscher –Ohlin (1933) developed a model with two goods, two factors of production and two countries. In their model the countries differ in their endowments, but they have the same technology for producing the goods. They predicted that countries will export products that intensively utilize the abundant factors of production and import products that intensively utilize the scarce factor. Vanek (1968) extends this for multi-good multifactor of production. Leontief (1953) tested the Hecksher- Ohlin model on the basis of U.S. data of exports and imports for 1947. He found that the empirical result is not consistent with the expected prediction based on the Hecksher- Ohlin model. Later on a wide range of explanations have been offered for this Leontief's paradox. One of these explanations is the fact that U.S. and foreign technologies are not the same.

Samuelson (1948) considered the case of two countries with same technology but with different factor endowments. According to him, if there is a free trade and both countries are producing both the goods and factor intensity reversals do not occur then factor prices are equalized across the countries. This is known as the factor price equalization theorem. This theorem is a remarkable result because it says that trade in goods is sufficient to get the same factor prices across the countries even though the factors of production are immobile across countries. Bardhan (1965) showed that when we have two goods, two factors of production, constant returns to scale and the technological change is happening in only one industry. In such a situation the output of the industry in which we have no change in technology will fall in absolute terms, even though the technological change is such that it saves the factor that is used less intensively in the changing industry provided absolute marginal product of less intensively used factor does not get reduced.

In this paper I am looking at the case when we have differences in technology across countries and the countries have different endowments. Even if we ensures that the factor intensity reversal is not happening then trade in goods is not enough to get the factor prices equalized across countries.

## Model

#### Assumptions

- (1) There are two countries say Home and Foreign.
- (2) There are two goods that are being produced by both countries and name the goods as *X* and *Y*.
- (3) We have Cobb-Douglas production function but the technology differs across goods and countries.
- (4) We have two factors of production labor (L) and capital (K).
- (5) Labor and capital are perfectly mobile within the country but they are immobile across different countries.
- (6) There is a free trade and we do not have any kind of barriers to trade like tariffs, quotas etc.
- (7) There are no transportation costs between the countries.
- (8) Perfect competition exists in the market in both the countries.
- (9) We have full employment in both the countries.

Now consider these notations

 $X_h$  = Good X produced by the country Home.

- $Y_h$  = Good Y produced by the country Home.
- $K_{xh}$  = Capital engaged in producing X in Home.
- $K_{yh}$  = Capital engaged in producing Y in Home.
- $L_{xh}$  = Labor engaged in producing X in Home.
- $L_{yh}$  = Labor engaged in producing Y in Home.
- $K_h$  = Total amount of capital available in Home.

 $L_h$  = Total amount of labor available for Home.

- $w_h$  = Wage rate in Home.
- $r_h$  = Interest rate in Home.
- $P_{xh}$  = Price of good X in Home.
- $P_{yh}$  = Price of good Y in Home.

For the country Foreign, we just replace all the h by the notation f.

Let us consider these equations which show the production functions for these two goods in two countries.

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 $0 \le \beta_f \le 1$ 

$$X_h = A \quad K_{xh}^{\alpha_h} L_{xh}^{1-\alpha_h} \tag{1} \qquad \qquad 0 < \alpha_h < 1 \qquad A > 0$$

$$Y_{h} = B \ K_{yh}^{\beta_{h}} L_{yh}^{1-\beta_{h}}$$
(2)  $0 < \beta_{h} < 1 \qquad B > 0$ 

- $K_{xh} + K_{yh} = K_h \tag{3}$
- $L_{xh} + L_{yh} = L_h \tag{4}$

$$X_f = A \quad K_{xf}^{\alpha_f} L_{xf}^{1-\alpha_f} \tag{5} \qquad 0 < \alpha_f <$$

$$Y_f = B \quad K_{yf}^{\beta_f} L_{yf}^{1-\beta_f} \tag{6}$$

$$K_{xf} + K_{yf} = K_f \tag{7}$$

$$L_{xf} + L_{yf} = L_f \tag{8}$$

Without trade case – We know that in equilibrium wage rate should be equal to price times the marginal product of labor and interest rate should be equal to price times marginal product of capital so we get the following equations.

$$w_h = P_{xh} A (1 - \alpha_h) \left( \frac{K_{xh}}{L_{xh}} \right)^{\alpha_h}$$
(9)

$$w_h = P_{yh} B(1 - \beta_h) \left(\frac{K_{yh}}{L_{yh}}\right)^{\beta_h}$$
(10)

$$r_h = P_{xh} A \alpha_h \left( \frac{K_{xh}}{L_{xh}} \right)^{\alpha_h - 1}$$
(11)

$$r_h = P_{yh} B \beta_h \left(\frac{K_{yh}}{L_{yh}}\right)^{\beta_h - 1}$$
(12)

Similarly for Foreign, we get these equations

$$w_f = P_{xf} A (1 - \alpha_f) \left( \frac{K_{xf}}{L_{xf}} \right)^{\alpha_f}$$
(13)

$$w_f = P_{yf} B (1 - \beta_f) \left( \frac{K_{yf}}{L_{yf}} \right)^{\beta_f}$$
(14)

$$r_f = P_{xf} A \alpha_f \left( \frac{K_{xf}}{L_{xf}} \right)^{\alpha_f - 1}$$
(15)

$$r_f = P_{yf} B \beta_f \left( \frac{K_{yf}}{L_{yf}} \right)^{\beta_f - 1}$$
(16)

By dividing equation (9) by equation (11) we get

$$\frac{w_h}{r_h} = \left(\frac{1-\alpha_h}{\alpha_h}\right) \left(\frac{K_{xh}}{L_{xh}}\right) \tag{17}$$

When we divide equation (10) by equation (12) we get

$$\frac{w_h}{r_h} = \left(\frac{1-\beta_h}{\beta_h}\right) \left(\frac{K_{xh}}{L_{xh}}\right) \tag{18}$$

From equation (17) and equation (18) we have

$$\left(\frac{1-\alpha_h}{\alpha_h}\right)\left(\frac{K_{xh}}{L_{xh}}\right) = \left(\frac{1-\beta_h}{\beta_h}\right)\left(\frac{K_{xh}}{L_{xh}}\right)$$
(19)

Similarly for Foreign we will have

$$\left(\frac{1-\alpha_f}{\alpha_f}\right)\left(\frac{K_{xf}}{L_{xf}}\right) = \left(\frac{1-\beta_f}{\beta_f}\right)\left(\frac{K_{xf}}{L_{xf}}\right)$$
(20)

Since  $\alpha_h, \beta_h, \alpha_f, \beta_f$  are constants so this negates the possibility of factor intensity reversal.

From equation (11) and (12), we get

$$\left(\frac{P_{xh}}{P_{yh}}\right) = \left(\frac{B\beta_h}{A\alpha_h}\right) \left(\frac{K_{xh}}{L_{xh}}\right)^{I-\alpha}{}_h \left(\frac{K_{yh}}{L_{yh}}\right)^{\beta}{}_h{}^{-I}$$

Equation (19) shows the relationship between capital- labor ratio of two industries in Home and by using that we can write price ratio as a function of capital-labor ratio of any of these two industries.

$$\left(\frac{P_{xh}}{P_{yh}}\right) = \left(\frac{B\beta_h}{A\alpha_h}\right) \left(\frac{1-\beta_h}{\beta_h}\right)^{l-\alpha} \left(\frac{\alpha_h}{1-\alpha_h}\right)^{l-\alpha} \left(\frac{K_{yh}}{L_{yh}}\right)^{\beta} \left(\frac{k_{yh}}{h}\right)^{\beta-\alpha} h$$
(21)

Similarly, for the Foreign we have,

$$\left(\frac{P_{xf}}{P_{yf}}\right) = \left(\frac{B\beta_f}{A\alpha_f}\right) \left(\frac{1-\beta_f}{\beta_f}\right)^{l-\alpha} f\left(\frac{\alpha_f}{1-\alpha_f}\right)^{l-\alpha} f\left(\frac{K_{yf}}{L_{yf}}\right)^{\beta_f - \alpha} f$$
(22)

Equation (18) gives us the relationship between  $(\frac{w_h}{r_h})$  and  $(\frac{K_{yh}}{L_{yh}})$ , using that we get the

relationship between product price-ratio and factor price-ratio, which is

$$\frac{P_{xh}}{P_{yh}} = \left(\frac{B\beta_h}{A\alpha_h}\right) \left(\frac{1-\beta_h}{\beta_h}\right)^{I-\alpha_h} \left(\frac{\alpha_h}{1-\alpha_h}\right)^{I-\alpha_h} \left(\frac{w_h}{r_h}\right)^{\beta_h-\alpha_h}$$
(23)

Similarly, for Foreign we will get,

$$\frac{P_{xf}}{P_{yf}} = \left(\frac{B\beta_f}{A\alpha_f}\right) \left(\frac{1-\beta_f}{\beta_f}\right)^{I-\alpha_f} \left(\frac{\alpha_f}{1-\alpha_f}\right)^{I-\alpha_f} \left(\frac{w_f}{r_f}\right)^{\beta_f - \alpha_f} (24)$$

From equation (19), we have  $\left(\frac{K_{xh}}{L_{xh}}\right) = \left(\frac{\alpha_h}{1-\alpha_h}\right) \left(\frac{1-\beta_h}{\beta_h}\right) \left(\frac{K_{yh}}{L_{yh}}\right)$ 

Now, if  $\alpha_h > \beta_h$ , then  $(\frac{K_{xh}}{L_{xh}}) > (\frac{K_{yh}}{L_{yh}})$  and if  $\alpha_h < \beta_h$ , then  $(\frac{K_{xh}}{L_{xh}}) < (\frac{K_{yh}}{L_{yh}})$ .

When  $\alpha_h = \beta_h$ , then we will have same technology for the production of both goods and capital-labor ratio will be same for both goods  $\left(\frac{K_{xh}}{L_{xh}}\right) = \left(\frac{K_{yh}}{L_{yh}}\right)$ .

Since total capital and labor is fixed for the country and  $\left(\frac{K_h}{L_h}\right)$  is a weighted average of

$$\left(\frac{K_{xh}}{L_{xh}}\right)$$
 and  $\left(\frac{K_{yh}}{L_{yh}}\right)$ .

, we must have  $\frac{K_h}{L_h}$  taking value somewhere between  $(\frac{K_{xh}}{L_{xh}})$  and  $(\frac{K_{yh}}{L_{yh}})$ .

If 
$$\alpha_h > \beta_h$$
, then  $\left(\frac{K_{xh}}{L_{xh}}\right) > \left(\frac{K_h}{L_h}\right) > \left(\frac{K_{yh}}{L_{yh}}\right)$ , if  $\alpha_h < \beta_h$ , then  $\left(\frac{K_{xh}}{L_{xh}}\right) < \left(\frac{K_h}{L_h}\right) < \left(\frac{K_{yh}}{L_{yh}}\right)$  will hold. When  
 $\alpha_h = \beta_h$ , we will get  $\left(\frac{K_{xh}}{L_{xh}}\right) = \left(\frac{K_h}{L_h}\right) = \left(\frac{K_{yh}}{L_{yh}}\right)$ .

Using equations (17) and (18), we can substitute  $\left(\frac{w_h}{r_h}\right)$  for capital-labor ratio in the two industries and get the range of values which  $\left(\frac{w_h}{r_h}\right)$  can take.

When 
$$\alpha_h > \beta_h$$
, then  $(\frac{1-\alpha_h}{\alpha_h})$   $(\frac{K_h}{L_h}) < (\frac{w_h}{r_h}) < (\frac{1-\beta_h}{\beta_h})$   $(\frac{K_h}{L_h})$  and if we have  $\alpha_h < \beta_h$ , then  $(\frac{1-\alpha_h}{\alpha_h})$ 

 $\left(\frac{K_h}{L_h}\right) > \left(\frac{w_h}{r_h}\right) > \left(\frac{1-\beta_h}{\beta_h}\right) \left(\frac{K_h}{L_h}\right)$ . In case we have same production function for both goods, then  $\alpha_h = \beta_h$  and  $\left(\frac{1-\alpha_h}{\alpha_h}\right) \left(\frac{K_h}{L_h}\right) = \left(\frac{w_h}{r_h}\right) = \left(\frac{1-\beta_h}{\beta_h}\right) \left(\frac{K_h}{L_h}\right)$ . Combining all these cases, we can write the range of values which  $\left(\frac{w_h}{r_h}\right)$  can take as

$$[\{1-\max (\alpha_{h}, \beta_{h})\}/\max (\alpha_{h}, \beta_{h})] (\frac{K_{h}}{L_{h}}) \leq \frac{w_{h}}{r_{h}} \leq [\{1-\min (\alpha_{h}, \beta_{h})\}/\min (\alpha_{h}, \beta_{h})] (\frac{K_{h}}{L_{h}})$$

$$(25)$$

We have the following equation for the relationship between total capital-labor ratio and the capital-labor ratio in two the production of two goods.

$$\frac{K_h}{L_h} = \frac{K_{xh}}{L_{xh}} \frac{L_{xh}}{L_h} + \frac{K_{yh}}{L_{yh}} \frac{L_{yh}}{L_h}$$
(26)  
$$\frac{K_h}{L_h} = \frac{K_{xh}}{L_{xh}} \frac{L_{xh}}{L_h} + \frac{K_{yh}}{L_{yh}} \left\{ 1 - \left(\frac{L_{xh}}{L_h}\right) \right\}$$
(27)

Using equation (17) and (18), we can get the relationship between  $\frac{K_h}{L_h}$  and  $\frac{w_h}{r_h}$ ,

$$\frac{K_h}{L_h} = \left(\frac{1-\alpha_h}{\alpha_h}\right) \left(\frac{w_h}{r_h}\right) \left(\frac{L_{xh}}{L_h}\right) + \left(\frac{1-\beta_h}{\beta_h} \left(\frac{w_h}{r_h}\right) \left\{1 - \left(\frac{L_{xh}}{L_h}\right)\right\}$$
(28)

The total capital-labor ratio  $\frac{K_h}{L_h}$  and the technological parameters  $(\alpha_h, \beta_h)$  together sets the range of value that factor price-ratio  $\frac{w_h}{r_h}$  can take, but in order to get the exact value of  $\frac{w_h}{r_h}$ , we also need to know  $L_{xh}$ . We have already derived the relationship between product price-ratio  $\frac{P_{xh}}{P_{yh}}$  and factor price-ratio  $\frac{w_h}{r_h}$ . We also know the relationship between factor price-ratio  $\frac{w_h}{r_h}$  and the capital-labor ratios  $\frac{K_{xh}}{L_{xh}}$  and  $\frac{K_{yh}}{L_{yh}}$ . We will get similar results for Foreign also, but once we allow for trade between Home and Foreign, the product price-ratio will get equalize and we will have one product price-ratio  $\frac{P_x}{P_y}$  for both countries.

We can have three different cases based on the differences in endowments and technology between Home and Foreign.

Case 1: Same technology ( $\alpha_h = \alpha_f, \beta_h = \beta_f$ ) in both countries but different capitallabor ratio  $\frac{K_h}{L_h} \neq \frac{K_f}{L_f}$ .

In this case, we will have a situation same as Hecksher-Ohlin theory of international trade and Samuelsons' factor price equalization theorem will hold. We can see this from equations (23) and (24), before trade the product price-ratio in two countries were

$$\frac{P_{xh}}{P_{yh}} = \left(\frac{B\beta_h}{A\alpha_h}\right) \left(\frac{1-\beta_h}{\beta_h}\right)^{l-\alpha} h \left(\frac{\alpha_h}{1-\alpha_h}\right)^{l-\alpha} h \left(\frac{w_h}{r_h}\right)^{\beta_h-\alpha} h$$

and

$$\frac{P_{xf}}{P_{yf}} = \left(\frac{B\beta_f}{A\alpha_f}\right) \left(\frac{1-\beta_f}{\beta_f}\right)^{I-\alpha_f} \left(\frac{\alpha_f}{1-\alpha_f}\right)^{I-\alpha_f} \left(\frac{w_f}{r_f}\right)^{\beta_f-\alpha_f}$$

for Home and Foreign respectively. After trade we will have same product price-ratio  $\frac{P_x}{P_y}$  for both countries and since the technology is same ( $\alpha_h = \alpha_{f,} \beta_h = \beta_f$ ), so after trade factor price-ratio will also be the same i.e.  $\frac{w_{ht}}{r_{ht}} = \frac{w_{ft}}{r_{ft}}$ , where t denotes after trade situation. The capital-labor ratio in the production of any particular good will be same for both Home and Foreign. Since the endowment differs across countries so we will have differences in the quantities of both goods produced by Home and Foreign.

Case 2 : Different technology  $(\alpha_h \neq \alpha_f, \beta_h \neq \beta_f)$  in both countries but same capitallabor ratio  $(\frac{K_h}{L_h}) = (\frac{K_f}{L_f})$ .

Since the endowments  $\frac{K_h}{L_h}$  and  $\frac{K_f}{L_f}$  play a role in the determination of range of the values which factor-price ratio can take before the trade, but after the trade product price-ratio get equalized and this product price-ratio  $\frac{P_x}{P_y}$  along with the technological parameters are going to determine the factor-price ratio. In this case, since the technology is different across two countries, so the factor price-ratio after trade will not get equalized i.e.  $\frac{w_{ht}}{r_{ht}} \neq \frac{w_{ft}}{r_{ft}}$ . For two countries, after trade we will get following relation between product price-ratio and factor price-ratio,

$$\frac{P_x}{P_y} = \left(\frac{B\beta_h}{A\alpha_h}\right) \left(\frac{1-\beta_h}{\beta_h}\right)^{l-\alpha}{}_h \left(\frac{\alpha_h}{1-\alpha_h}\right)^{l-\alpha}{}_h \left(\frac{w_{ht}}{r_{ht}}\right)^{\beta_h-\alpha}{}_h$$
(29)

$$\frac{P_x}{P_y} = \left(\frac{B\beta_f}{A\alpha_f}\right) \left(\frac{1-\beta_f}{\beta_f}\right)^{l-\alpha_f} \left(\frac{\alpha_f}{1-\alpha_f}\right)^{l-\alpha_f} \left(\frac{w_{ft}}{r_{ft}}\right)^{\beta_f-\alpha_f} f$$
(30)

From the two equations given above, it is clear that in general we will have  $\frac{w_{ht}}{r_{ht}} \neq \frac{w_{ft}}{r_{ft}}$ , and we can get  $\frac{w_{ht}}{r_{ht}} = \frac{w_{ft}}{r_{ft}}$  if and only if  $\frac{P_x}{P_y}$  takes a particular value not otherwise. The capital-labor ratio in the production of same good will also differ across countries, this we can say because we have already derived a relationship between factor price-ratio and capital-labor ratio. Equation (21) and (22) shows the relationship between product price-ratio and capital-labor ratio which involves

the technological parameters  $\alpha_h$ ,  $\beta_h$  and  $\alpha_f$ ,  $\beta_f$ . Since we have ( $\alpha_h \neq \alpha_f$ ,  $\beta_h \neq \beta_f$ ), we will get different capital-labor ratio across countries. Even if the production function is different for one good in both countries, while it is same for the other good all our results will hold. In this case the endowments are going to affect the amount of final goods produced by the countries, but with same endowment and different technology the countries will end up producing different quantities of both goods.

Case 3: Different technology  $(\alpha_h \neq \alpha_f, \beta_h \neq \beta_f)$  in both countries and different capitallabor ratio  $\frac{K_h}{L_h} \neq \frac{K_f}{L_f}$ .

In this case we will get the same result as in case 2 and all the arguments given above will hold.

Case 4: Different scale parameters  $(A_h \neq A_f, B_h \neq B_f)$  in both countries and  $(\alpha_h = \alpha_f, \beta_h = \beta_f)$ .

In this case the factor price ratio will not get equalized after trade as we can see from equations (29) and (30).

#### **Case 5: Perfect capital mobility across both countries.**

If we allow for the perfect capital mobility across both countries the interest rate will get equalized after trade but the wage rate will continue to differ between Home and Foreign even after trade.

# Case 6 : Technology differs across countries only for one good $(\alpha_h \neq \alpha_f, \beta_h = \beta_f)$ .

When we consider a situation in which the technology differs for one good and for other good it is same in both countries, trade will not result in factor price equalization. We can clearly see this from equations (29) and (30).

#### Conclusion

We looked at the case of trade with differences in technology across two countries. We also allow for the differences in the relative endowment of capital and labor. In autarky, endowment and technology and the combinations of two goods produced are going to determine factor price-ratio, goods price-ratio and capital-labor ratio used in the production of two goods. After trade, the goods price-ratio will get equalized, and this product price-ratio along with the technology are going to determine factor price-ratio and capital-labor ratio. Endowments are going to affect only the quantities of goods being produced. We looked at three possible cases in which we have differences in either technology or endowments or both. Factor price equalization theorem says that with the same technology and different endowments, trade in goods will equalize the factor prices across the countries, provided we don't have factor intensity reversal. In this paper, we derive that result, but we also consider the cases in which technology differs across countries. When we have differences in technology across countries, after trade, even though the goods price-ratio gets equalized, it is not going to equalize factor price-ratio.

Trade will not result in factor price equalization when the technology differs only for one good or when the countries differ in the scale parameters. When we allow capital to be perfectly mobile across countries, wage rate will continue to differ across countries even after trade. It is also possible to have a situation in which factor-price difference between countries will increase as a result of trade. We can also have a situation in which a capital abundant country is going to export a labor intensive good. It is also possible to have same product factor price ratio and different product price ratio across countries before the trade but after trade we can have different factor price ratio and same product price ratio.

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