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PROSPECTIVE CONTROL IN AN ORGANIZATION THROUGH TWO GRADE SYSTEMS

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ABSTRACT

In this paper a two grade organization in which depletion of manpower occurs due to its policy decisions is considered. It is assumed that the loss of manpower is linear. Let X_i a random variable is the loss of manpower due to the i^{th} decision epochs, forming a sequence of i.i.d random variables. The expected time and variance of the time to recruitment are obtained for the models under special conditions. The analytical results are numerically illustrated with conclusions.

Keywords: expected time, grade, organization, and threshold.

INTRODUCTION

Consider an organization having two grades in which the depletion of manpower occurs at every decision epoch. Exits of personnel which is in other words known as wastage is an important aspect in manpower planning. Many models have been discussed using different types of distributions. Such models could be seen in Gupta, R. D (1999) and Gupta, R. D. (2001). Have

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obtained the expected time to recruitment times are independent and identically distributed random variables. In the authors have obtained the mean and variance of the time to recruitment for a two graded manpower system when the thresholds for the loss of manpower in the two grades are combined together. Esary et al. (1973), consider a system, which can be either an engineering system or a bio-system, subjected to shocks occurring randomly in time. One can see for more detail in Pandiyan et al. (2012), Kannadasan et al. (2012) and Sathiyamoorthi (1980) about the expected time to cross the threshold level of the organization.

These assumptions are somewhat artificial, but are made because of the lack of detailed real-world information on one hand and in order to illustrate the proceedings on the other hand. The organization comprises two grades of personnel. Mobility or transfer of manpower from one grade to the other is permitted. The time to recruitment is equal to the maximum of the time taken for each one of the two grades to cross the threshold which follows three parameter generalized exponential distribution. The processes which give rise to policy revisions and the threshold random variables are statistically independent. The policy decisions are taken with inter arrival times which are i.i.d. random variables depending upon the market environment, production, etc.

NOTATIONS

X_i : a continuous random variable denoting the amount of loss of manpower caused to the system on the ith occasion of policy announcement (Shock) 1, 2….k and X_i's are i.i.d

Y_1, Y_2 : continuous random variable denoting the threshold levels for the two grades which follows three parameter generalized exponential distribution.

U_i : a random variable denoting the inter-arrival times between contact with c.d.f. $F_i(.)$, $i = 1,2,3 \dots k$.

$g(.)$: The probability density functions of X_i. $g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: the k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between successive policy announcement with the corresponding c.d.f. $F(.)$.

$F_k(.)$: k-fold convolution of $F(.)$. $S(.)$: Survival function.

$V_k(t)$: Probability of exactly k policy announcements. $L(t)$: $1 - S(t)$.

MODELS DESCRIPTION

Consider an organization with two grades taking decisions at random epochs in [0,∞), at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quit. It is assumed that the loss of manpower is linear and cumulative.

$$\bar{H}(x) = \left\{e^{-(\lambda_2 x - \lambda_2 \theta_2)} + e^{-\lambda_1(x-\theta_1)} - e^{-\lambda_1(x-\theta_1)-\lambda_2(x-\theta_2)}\right\} \tag{1}$$

$$P(X_i < Y) = \int_0^\infty g_k(x)\bar{H}(x)dx$$

$$= \int_0^\infty g^*(x)\left\{e^{-(\lambda_2 x - \lambda_2 \theta_2)} + e^{-\lambda_1(x-\theta_1)} - e^{-\lambda_1(x-\theta_1)-\lambda_2(x-\theta_2)}\right\}dx \tag{2}$$

$$= \left[g^*[\lambda_2(1-\theta_2)]\right]^k + \left[g^*[\lambda_1(1-\theta_1)]\right]^k - \left[g^*[\lambda_1(1-\theta_1)] + [\lambda_2(1-\theta_2)]\right]^k$$

$P(T > t) =$ Probability that exactly k decision epochs in (0,t] and the combined threshold level is not crossed

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(X_i < Y)$$

Therefore on simplification it can be shown that

$L(t) = 1 - s(t)$

Taking laplace transformation of L(T),we get

$$L(t) = 1 - \left\{\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*[\lambda_2(1-\theta_2)]\right]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*[\lambda_1(1-\theta_1)]\right]^k - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[g^*[\lambda_1(1-\theta_1)[\lambda_2(1-\theta_2)]]\right]^k\right\}$$

$$L(t) = 1 - \Bigg\{ [1 - g^*[\lambda_2(1-\theta_2)]] \sum_{K=1}^{\infty} F_k(t) [g^*[\lambda_2(1-\theta_2)]]^{k-1} + 1$$
$$- [1 - g^*[\lambda_1(1-\theta_1)]] \sum_{K=1}^{\infty} [g^*[\lambda_1(1-\theta_1)]]^{k-1} - 1$$
$$+ [-g^*[\lambda_1 + \lambda_2(1-\theta_2)]] \sum_{K=1}^{\infty} F_k(t) [g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]]^{k-1} \Bigg\} \quad (3)$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3) we get

$$L^*(t) = \frac{[1 - g^*[\lambda_2(1-\theta_2)]]c}{[c + s - g^*[\lambda_2(1-\theta_2)]c]} + \frac{[1 - g^*[\lambda_1(1-\theta_1)]]c}{[c + s - g^*[\lambda_1(1-\theta_1)]c]}$$
$$- \frac{[1 - g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]]c}{[c + s - g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]\, c]} \quad (4)$$

$$E(T) = -\frac{d}{dt} l^*(t) \text{ given } t = 0$$

$$= \frac{c\,[1 - g^*[\lambda_2(1-\theta_2)]]}{[c + s - g^*[\lambda_2(1-\theta_2)]c]^2} + \frac{c\,[1 - g^*[\lambda_1(1-\theta_1)]]}{[c + s - g^*[\lambda_1(1-\theta_1)]c]^2}$$
$$- \frac{c\,[1 - g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]]}{[c + s - g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]\, c]^2} \qquad \text{given } s = 0$$

$$= \frac{1}{c\,[1 - g^*[\lambda_2(1-\lambda_2\theta_2)]]} + \frac{1}{c\,[1 - g^*[\lambda_1(1-\lambda_1\theta_1)]]}$$
$$- \frac{1}{c\,[1 - g^*[\lambda_1(1-\lambda_1\theta_1) + \lambda_2(1-\lambda_2\theta_2)]]} \quad (5)$$

$$E(T^2) = -\frac{d^2}{ds^2} l^*(s) \text{ given } s = 0$$

We know that

$$= \frac{2}{c^2[1 - g^*[\lambda_2(1-\theta_2)]]^2} + \frac{2}{c^2[1 - g^*[\lambda_1(1-\theta_1)]]^2} - \frac{2}{c^2[1 - g^*[\lambda_1(1-\theta_1) + \lambda_2(1-\theta_2)]]^2}$$

$g^*(.) \sim \exp(\mu)$, $g^*(\lambda) \sim \exp\left(\frac{\mu}{\mu+\lambda}\right)$, $g^*(\lambda\theta) \sim \exp\left(\frac{\mu}{\mu+\lambda\theta}\right)$,

$$E(T) = \frac{1}{c\left[1-\left(\frac{\mu}{\mu+\lambda_2}-\frac{\mu}{\mu+\lambda_{2\theta_2}}\right)\right]}\frac{1}{c\left[1-\left(\frac{\mu}{\mu+\lambda_1}-\frac{\mu}{\mu+\lambda_{1\theta_1}}\right)\right]}\frac{1}{c\left[1-\left(\frac{\mu}{\mu+\lambda_2}-\frac{\mu}{\mu+\lambda_{2\theta_2}}+\frac{\mu}{\mu+\lambda_{1\theta_1}}\right)\right]}$$

$$E(T) = \frac{\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2}{c\,[\mu^2+2\,\lambda_2\mu+\lambda_2^2\theta_2]}+\frac{\mu^2+\lambda_1\mu\,\theta_1+\lambda_1\mu+\lambda_1^2\theta_1}{c\,[\mu^2+\;2\lambda_1\mu+\lambda_1^2\theta_1]}$$

$$-\frac{(\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2)(\mu^2+\;\lambda_1\mu\theta_1+\;\lambda_1\mu+\lambda_1^2\theta_1)}{c[\mu^4+2\mu^3\lambda_1+\mu^2\lambda_1^2\theta_1+2\mu^3\lambda_2+\lambda_1\lambda_2\mu^2\theta_1+3\lambda_1\lambda_2\mu^2+\lambda_2^2\theta_2\mu^2+2\lambda_1\lambda_2^2\,\mu\,\theta_2+\lambda_1^2\lambda_2^2\theta_1\theta_2+\lambda_1\lambda_2\mu^2\theta_2]}$$

$$V(T) = E(T^2)-[E(T)]^2$$

On simplification it can be shown that

$$V(T) = \left(\frac{\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2}{c\,[\mu^2+2\,\lambda_2\mu+\lambda_2^2\theta_2]}\right)^2+\left(\frac{\mu^2+\lambda_1\mu\,\theta_1+\lambda_1\mu+\lambda_1^2\theta_1}{c\,[\mu^2+\;2\lambda_1\mu+\lambda_1^2\theta_1]}\right)^2$$

$$-\left(\frac{(\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2)(\mu^2+\;\lambda_1\mu\theta_1+\;\lambda_1\mu+\lambda_1^2\theta_1)}{c[\mu^4+2\mu^3\lambda_1+\mu^2\lambda_1^2\theta_1+2\mu^3\lambda_2+\lambda_1\lambda_2\mu^2\theta_1+3\lambda_1\lambda_2\mu^2+\lambda_2^2\theta_2\mu^2+2\lambda_1\lambda_2^2\,\mu\,\theta_2+\lambda_1^2\lambda_2^2\theta_1\theta_2+\lambda_1\lambda_2\mu^2\theta_2]}\right)^2$$

$$-2\left(\frac{\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2}{c\,[\mu^2+2\,\lambda_2\mu+\lambda_2^2\theta_2]}\right)\left(\frac{\mu^2+\lambda_1\mu\,\theta_1+\lambda_1\mu+\lambda_1^2\theta_1}{c\,[\mu^2+\;2\lambda_1\mu+\lambda_1^2\theta_1]}\right)+2\left(\frac{\mu^2+\lambda_1\mu\,\theta_1+\lambda_1\mu+\lambda_1^2\theta_1}{c\,[\mu^2+\;2\lambda_1\mu+\lambda_1^2\theta_1]}\right)$$

$$\left(\frac{(\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2)(\mu^2+\;\lambda_1\mu\theta_1+\;\lambda_1\mu+\lambda_1^2\theta_1)}{c[\mu^4+2\mu^3\lambda_1+\mu^2\lambda_1^2\theta_1+2\mu^3\lambda_2+\lambda_1\lambda_2\mu^2\theta_1+3\lambda_1\lambda_2\mu^2+\lambda_2^2\theta_2\mu^2+2\lambda_1\lambda_2^2\,\mu\,\theta_2+\lambda_1^2\lambda_2^2\theta_1\theta_2+\lambda_1\lambda_2\mu^2\theta_2]}\right)$$

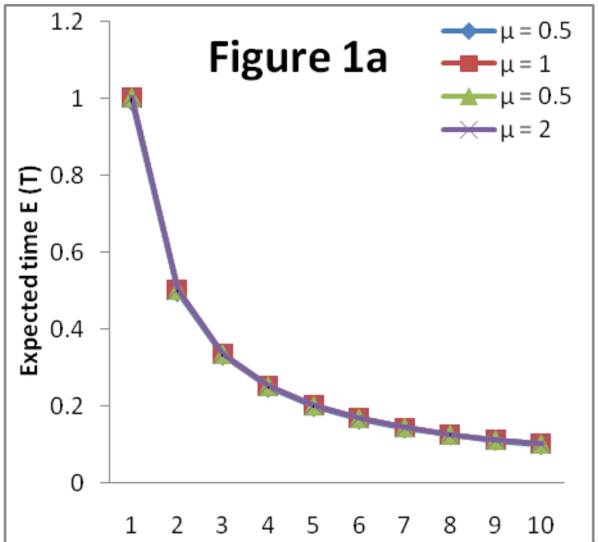
$$+2\left(\frac{\mu^2+\lambda_1\mu\,\theta_1+\lambda_1\mu+\lambda_1^2\theta_1}{c\,[\mu^2+\;2\lambda_1\mu+\lambda_1^2\theta_1]}\right)$$

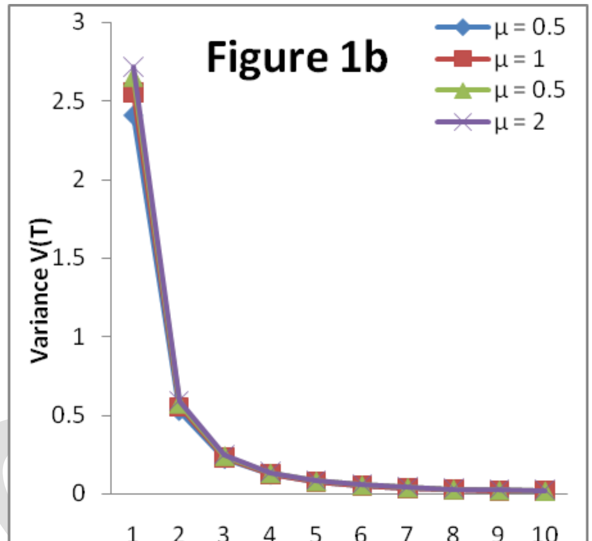
$$\left(\frac{(\mu^2+\lambda_2\mu\,\theta_2+\lambda_2\mu+\lambda_2^2\theta_2)(\mu^2+\;\lambda_1\mu\theta_1+\;\lambda_1\mu+\lambda_1^2\theta_1)}{c[\mu^4+2\mu^3\lambda_1+\mu^2\lambda_1^2\theta_1+2\mu^3\lambda_2+\lambda_1\lambda_2\mu^2\theta_1+3\lambda_1\lambda_2\mu^2+\lambda_2^2\theta_2\mu^2+2\lambda_1\lambda_2^2\,\mu\,\theta_2+\lambda_1^2\lambda_2^2\theta_1\theta_2+\lambda_1\lambda_2\mu^2\theta_2]}\right)$$

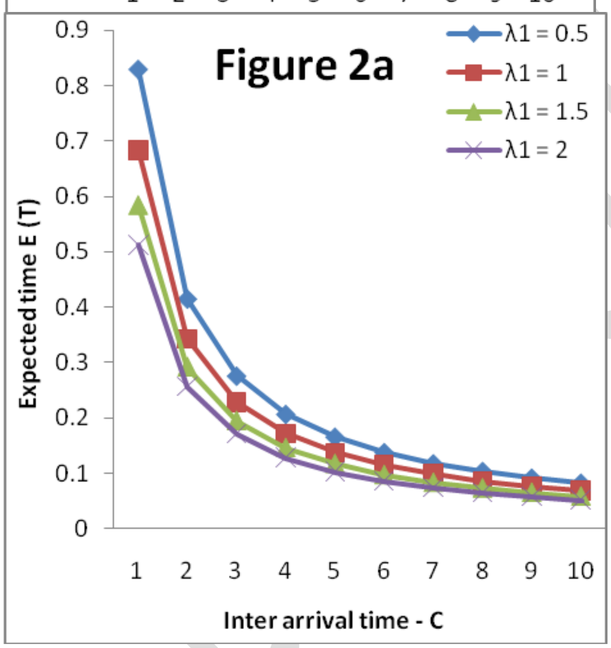
The mean and variance of the time to recruitment for the present model.

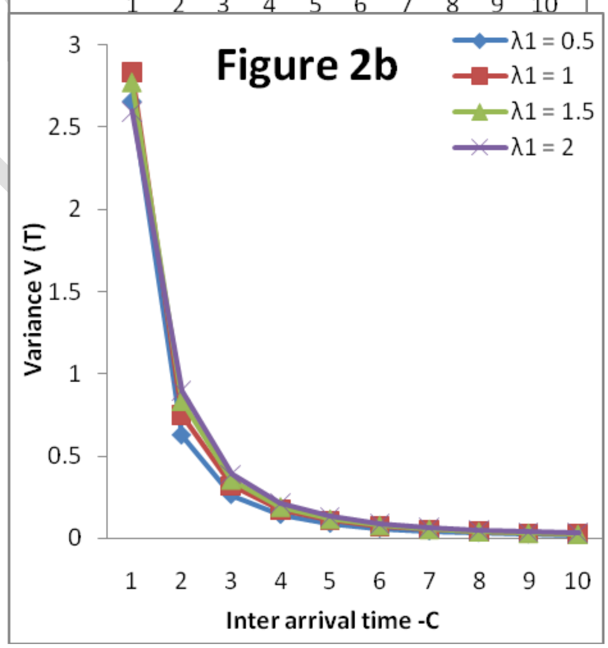
NUMERICAL ILLUSTRATIONS

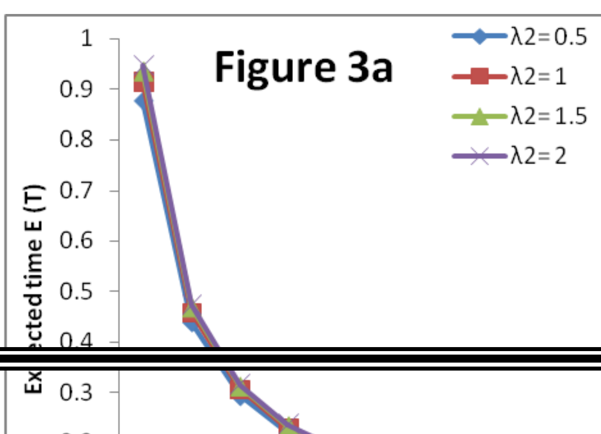
The influence of parameters on the performance measures namely the mean and variance of the time to recruitment are studied numerically. In the following table these performance measures are calculated by varying the parameters one at a time and keeping the parameters $\mu, \lambda_1, \lambda_2, \theta_1$ and θ_2 fixed.

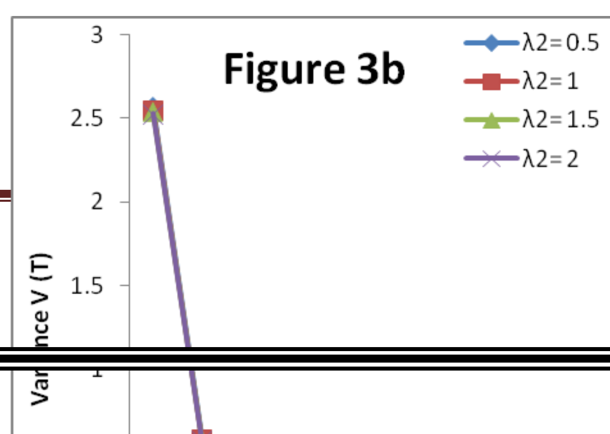












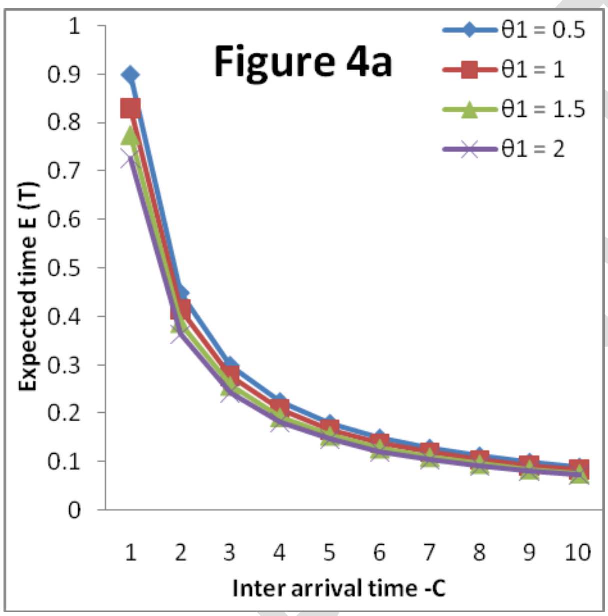


Figure 4a
θ1 = 0.5
θ1 = 1
θ1 = 1.5
θ1 = 2
Expected time E (T)
Inter arrival time -C

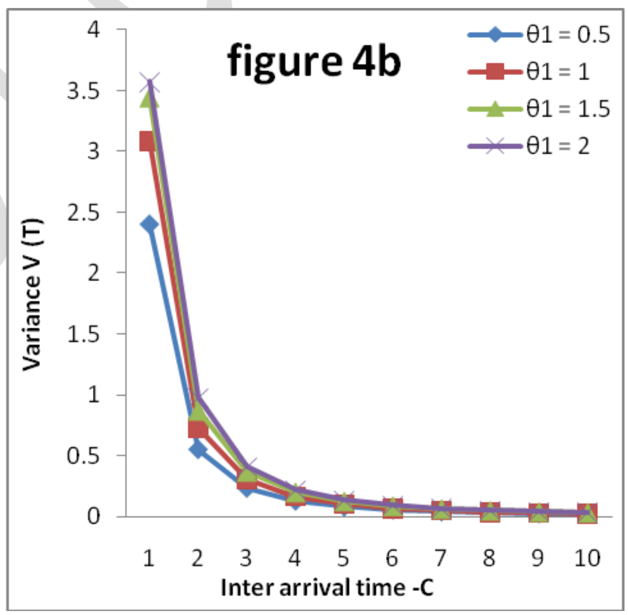


figure 4b
θ1 = 0.5
θ1 = 1
θ1 = 1.5
θ1 = 2
Variance V (T)
Inter arrival time -C

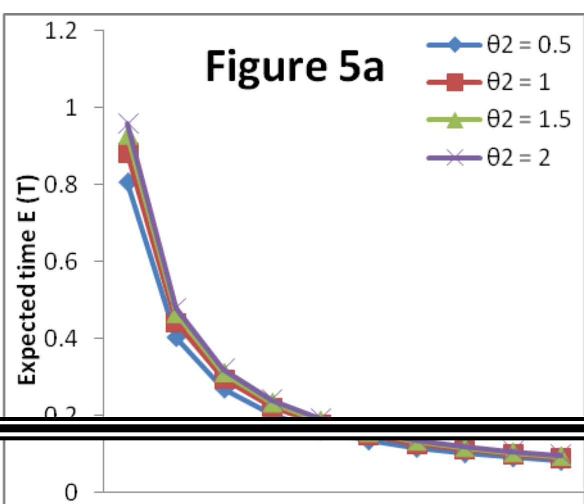


Figure 5a
θ2 = 0.5
θ2 = 1
θ2 = 1.5
θ2 = 2
Expected time E (T)

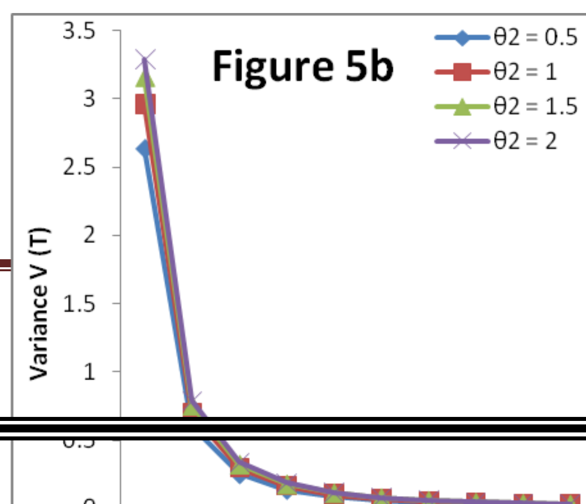


Figure 5b
θ2 = 0.5
θ2 = 1
θ2 = 1.5
θ2 = 2
Variance V (T)

CONCLUSIONS

When μ is kept fixed with other parameters $\lambda_1, \lambda_2, \theta_1$ and θ_2 the inter-arrival time 'c', which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold of recruitment is decreasing, for all cases of the parameter value $\mu = 0.5,1,1.5,2.$ when the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1a, and 1b. The same case is found in Variance V (T) which is observed in Figure 1a, and1b.

When$\lambda_1$is kept fixed with other parameters $\mu, \lambda_2, \theta_1$and θ_2 the inter-arrival time 'c' increases, the value of the expected time E(T) to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value $\lambda_1 = 0.5,1,1.5, 2.$ When the value of the parameter λ_1 increases, the expected time is found increasing. This is indicated in Figure 2a and 2b. The same case is observed in the threshold of recruitment of Variance V(T) which is observed in Figure 2a and 2b.

When λ_2 is kept fixed with other parameters μ, λ_1, θ_1 and θ_2 the inter-arrival time 'c' increases, the value of the expected time E (T) to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value$\theta = 0.5,1,1.5, 2.$ When the value of the parameter θ increases, the expected time is found increasing. This is indicated in Figure 3a and 3b. The same case is observed in the threshold of recruitment of Variance V (T) which is observed in Figure 3a and 3b.

When θ_1 is kept fixed with other parameters $\mu, \lambda_1, \lambda_2$ and θ_2 the inter-arrival time 'c' increases, the value of the expected time E(T) to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value $\theta_1 = 0.5, 1, 1.5, 2.$ When the value of the parameter θ_1 increases, the expected time is found increasing. This is indicated in Figure 4a and 4b. The same case is observed in the threshold of recruitment of Variance V(T) which is observed in Figure 4a and 4b.

When θ_2 is kept fixed with other parameters $\mu, \lambda_1, \lambda_2$ and θ_1 the inter-arrival time 'c' increases, the value of the expected time E(T) to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value $\theta_2 = 0.5, 1, 1.5, 2.$ When the value of the parameter θ_2 increases, the expected time is found increasing. This is indicated in Figure 4a and 4b. The same case is observed in the threshold of recruitment of Variance V(T) which is observed in Figure 5a and 5b.

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